Radicals Lesson 5
Division with Radical Expressions

Important Note

For all braille examples, emboss the “L5-Radicals-Problems-Only.brf” file as a supplement to this lesson.

Background

After completing “Lesson 1 Radical Expressions” and “Lesson 2 Radical Expressions with an Index,” you are ready to learn how to read and write the Nemeth Code used in dividing and simplifying **radical expressions**.

As a quick review, when writing a **square root**, you follow three simple steps. You would braille:

1. The radical symbol (dots 3-4-5) ($\sqrt{}$) ⠜
2. The radicand, value inside/under a radical symbol, which you want to find the root of
3. The termination indicator (dots 1-2-4-5-6) ⠻

The following steps outline how to write the principal square root of 4:

1. Radical symbol (dots 3-4-5) ($\sqrt{}$) ⠜
2. Four (dots 2-5-6) ⠲
3. Termination indicator (dots 1-2-4-5-6) ⠻

$\sqrt{4}$

⠜⠲⠻

When writing a radical with an index, you follow these simple steps. You would braille:

1. The index-of-radical indicator (dots 1-2-6) ⠣
2. The index of the radical
3. The radical symbol (dots 3-4-5) ⠜
4. The radicand, value inside/under a radical symbol, which you want to find the root of

5. The termination indicator (dots 1-2-4-5-6) ⠻

The following steps outline how to write the cube root of 27:

1. Index-of-radical indicator (dots 1-2-6) ⠣
2. Three (dots 2-5) ⠒
3. Radical symbol (dots 3-4-5) ⠜
4. Twenty-seven (dots 2-3, dots 2-3-5-6) ⠆⠶
5. Termination indicator (dots 1-2-4-5-6) ⠻

$\sqrt[3]{27}$

⠣⠒⠜⠆⠶⠻

All fractions in this lesson use a horizontal fraction line.

Basic Rules for Simplifying Radicals with Quotients

For any natural number index k and any real numbers a and b, b not equal to 0, where the kth root of a and the kth root of b are real numbers, the kth root of open fraction a over b close fraction end root equals open fraction the kth root of a end root over the kth root of b end root close fraction.

$$\sqrt[k]{\frac{a}{b}} = \frac{\sqrt[k]{a}}{\sqrt[k]{b}}$$

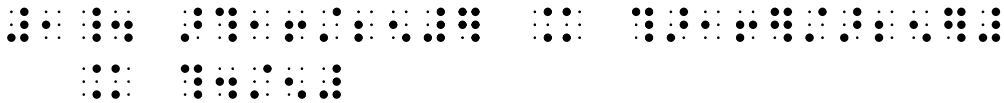
This theorem is used extensively when simplifying radical expressions involving quotients. Follow the same conventions for reading and spacing as we did in Lessons 1 to 4.

Examples of Simplifying Radicals with Quotients

In the examples below, when you hear the word times, only use a multiplication dot when specifically indicated.

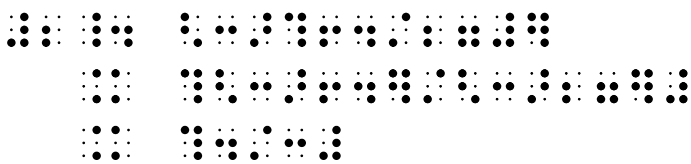
1. The square root of open fraction sixteen over twenty-five close fraction end root equals open fraction the square root of sixteen end root over the square root of twenty-five end root close fraction equals four-fifths.

$$\sqrt{\frac{16}{25}}=\frac{\sqrt{16}}{\sqrt{25}}=\frac{4}{5}$$



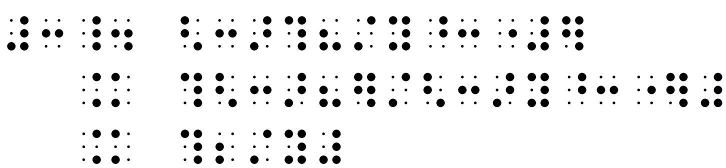
2. The cube root of open fraction sixty-four over twenty-seven close fraction end root equals open fraction the cube root of sixty-four end root over the cube root of twenty-seven end root close fraction equals four-thirds.

$$\sqrt[3]{\frac{64}{27}}=\frac{\sqrt[3]{64}}{\sqrt[3]{27}}=\frac{4}{3}$$



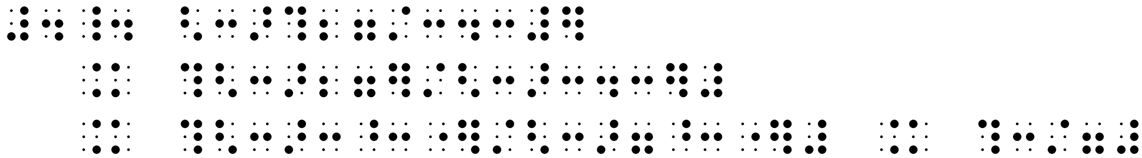
3. The cube root of open fraction eight over y cubed close fraction end root equals open fraction the cube root of eight end root over the cube root of y cubed end root close fraction equals open fraction two over y close fraction.

$$\sqrt[3]{\frac{8}{y^3}}=\frac{\sqrt[3]{8}}{\sqrt[3]{y^3}}=\frac{2}{y}$$



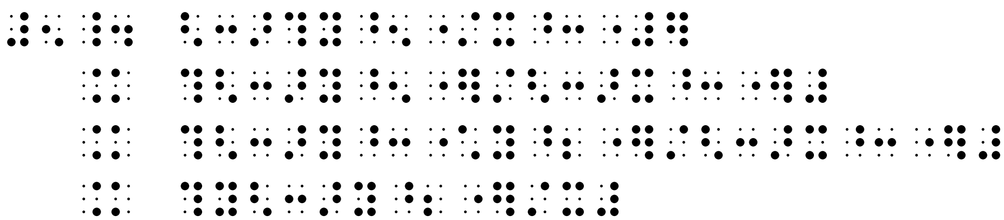
4. The cube root of open fraction twenty-seven over three hundred forty-three close fraction end root equals open fraction the cube root of twenty-seven end root over the cube root of three hundred forty-three end root close fraction equals open fraction the cube root of three cubed end root over the cube root of seven cubed end root close fraction equals three-sevenths.

$$\sqrt[3]{\frac{27}{343}} = \frac{\sqrt[3]{27}}{\sqrt[3]{343}} = \frac{\sqrt[3]{3^3}}{\sqrt[3]{7^3}} = \frac{3}{7}$$



5. The cube root of open fraction y to the fifth power over x cubed close fraction end root equals open fraction the cube root of y to the fifth power end root over the cube root of x cubed end root close fraction equals open fraction the cube root of y cubed times (multiplication dot) y squared end root over the cube root of x cubed end root close fraction equals open fraction y cube root of y squared end root over x close fraction.

$$\sqrt[3]{\frac{y^5}{x^3}} = \frac{\sqrt[3]{y^5}}{\sqrt[3]{x^3}} = \frac{\sqrt[3]{y^3 \cdot y^2}}{\sqrt[3]{x^3}} = \frac{y\sqrt[3]{y^2}}{x}$$



Activity Time for Simplifying Radicals with Quotients

Write the problems from Examples 1 to 5.

1. The square root of open fraction sixteen over twenty-five close fraction end root equals open fraction the square root of sixteen end root over the square root of twenty-five end root close fraction equals four-fifths.
2. The cube root of open fraction sixty-four over twenty-seven close fraction end root equals open fraction the cube root of sixty-four end root over the cube root of twenty-seven end root close fraction equals four-thirds.
3. The cube root of open fraction eight over y cubed close fraction end root equals open fraction the cube root of eight end root over the cube root of y cubed end root close fraction equals open fraction two over y close fraction.

4. The cube root of open fraction twenty-seven over three hundred forty-three close fraction end root equals open fraction the cube root of twenty-seven end root over the cube root of three hundred forty-three end root close fraction equals open fraction the cube root of three cubed end root over the cube root of seven cubed end root close fraction equals three-sevenths.
5. The cube root of open fraction y to the fifth power over x cubed close fraction end root equals open fraction the cube root of y to the fifth power end root over the cube root of x cubed end root close fraction equals open fraction the cube root of y cubed times (multiplication dot) y squared end root over the cube root of x cubed end root close fraction equals open fraction y cube root of y squared end root over x close fraction.

Basic Rules for Dividing Radical Expressions

Reversing the equation of the theorem above, for any natural number index k and any real numbers a and b, b not equal to 0, where the kth root of a and the kth root of b are real numbers, open fraction the kth root of a end root over the kth root of b end root close fraction equals the kth root of open fraction a over b close fraction end root.

$$\frac{\sqrt[k]{a}}{\sqrt[k]{b}} = \sqrt[k]{\frac{a}{b}}$$

This theorem is used extensively when dividing radical expressions, as shown below. Follow the same conventions for reading and spacing as we did in Lessons 1 to 4.

Examples for Dividing Radical Expressions

In the examples below, when you hear the word times, only use a multiplication dot when specifically indicated.

1. Open fraction the square root of ninety-six end root over the square root of six end root close fraction equals the square root of open fraction ninety-six over six close fraction end root equals the square root of sixteen end root equals four.

$$\frac{\sqrt{96}}{\sqrt{6}} = \sqrt{\frac{96}{6}} = \sqrt{16} = 4$$

⠼⠂⠸⠲⠀⠹⠜⠔⠖⠻⠌⠜⠖⠻⠼⠀⠨⠅⠀⠜⠹⠔⠖⠌⠖⠼⠻
⠀⠀⠨⠅⠀⠜⠂⠖⠻⠀⠨⠅⠀⠼⠲

2. Open fraction four cube root of thirty-two end root over the cube root of two end root close fraction equals four cube root of open fraction thirty-two over two close fraction end root equals four cube root of sixteen end root.

$$\frac{4\sqrt[3]{32}}{\sqrt[3]{2}} = 4\sqrt[3]{\frac{32}{2}} = 4\sqrt[3]{16}$$

⠼⠆⠸⠲⠀⠹⠲⠣⠒⠜⠒⠆⠻⠌⠣⠒⠜⠆⠻⠼
⠀⠀⠨⠅⠀⠼⠲⠣⠒⠜⠹⠒⠆⠌⠆⠼⠻⠀⠨⠅⠀⠼⠲⠣⠒⠜⠂⠖⠻

3. Four cube root of sixteen end root equals four cube root of eight times (multiplication dot) two end root equals four cube root of two cubed times (multiplication dot) two end root equals eight cube root of two end root.

Example 3 completes the simplification of Example 2 and serves as a review.

$$4\sqrt[3]{16} = 4\sqrt[3]{8 \cdot 2} = 4\sqrt[3]{2^3 \cdot 2} = 8\sqrt[3]{2}$$

⠼⠒⠸⠲⠀⠼⠲⠣⠒⠜⠂⠖⠻⠀⠨⠅⠀⠼⠲⠣⠒⠜⠦⠡⠆⠻
⠀⠀⠨⠅⠀⠼⠲⠣⠒⠜⠆⠘⠒⠐⠡⠆⠻⠀⠨⠅⠀⠼⠦⠣⠒⠜⠆⠻

4. Open fraction twelve times the square root of one hundred twenty-eight x y end root over two times the square root of two end root close fraction equals six times the square root of open fraction one hundred twenty-eight x y over two close fraction end root equals six times the square root of sixty-four x y end root.

$$\frac{12\sqrt{128xy}}{2\sqrt{2}} = 6\sqrt{\frac{128xy}{2}} = 6\sqrt{64xy}$$

⠼⠲⠸⠲⠀⠹⠂⠆⠜⠂⠆⠦⠭⠽⠻⠌⠆⠜⠆⠻⠼
⠀⠀⠨⠅⠀⠼⠖⠜⠹⠂⠆⠦⠭⠽⠌⠆⠼⠻⠀⠨⠅⠀⠼⠖⠜⠖⠲⠭⠽⠻

5. Six square root of sixty-four x y end root equals six square root of eight squared x y end root equals six times (multiplication dot) eight square root of x y end root equals forty-eight square root of x y end root.

   Example 5 completes the simplification of Example 4 and serves as a review.

   $$6\sqrt{64xy} = 6\sqrt{8^2 xy} = 6 \cdot 8\sqrt{xy} = 48\sqrt{xy}$$

   ⠼⠢⠸⠲ ⠼⠖⠜⠖⠲⠭⠽⠻ ⠨⠅ ⠼⠖⠜⠦⠘⠆⠐⠭⠽⠻
   ⠨⠅ ⠼⠖⠡⠦⠜⠭⠽⠻ ⠨⠅ ⠼⠲⠦⠜⠭⠽⠻

Activity Time for Dividing Radical Expressions

Write the five problems from Examples 1 to 5.

1. Open fraction the square root of ninety-six end root over the square root of six end root close fraction equals the square root of open fraction ninety-six over six close fraction end root equals the square root of sixteen end root equals four.
2. Open fraction four cube root of thirty-two end root over the cube root of two end root close fraction equals four cube root of open fraction thirty-two over two close fraction end root equals four cube root of sixteen end root.
3. Four cube root of sixteen end root equals four cube root of eight times (multiplication dot) two end root equals four cube root of two cubed times (multiplication dot) two end root equals eight cube root of two end root.
4. Open fraction twelve times the square root of one hundred twenty-eight x y end root over two times the square root of two end root close fraction equals six times the square root of open fraction one hundred twenty-eight x y over two close fraction end root equals six times the square root of sixty-four x y end root.
5. Six square root of sixty-four x y end root equals six square root of eight squared x y end root equals six times (multiplication dot) eight square root of x y end root equals forty-eight square root of x y end root.