Radicals Lesson 4
Multiplying and Simplifying
Radical Expressions

Important Note

For all braille examples, emboss the "L4-Radicals-Problems-Only.brf" file as a supplement to this lesson.

Background

After completing "Lesson 1 Radical Expressions" and "Lesson 2 Radical Expressions with an Index," you are ready to learn how to read and write the Nemeth Code used in multiplying and simplifying **radical expressions**.

As a quick review, when writing a **square root**, you follow three simple steps. You would braille:

1. The radical symbol (dots 3-4-5) $(\sqrt{\ \ })$ ⠜
2. The radicand, value inside/under a radical symbol, which you want to find the root of
3. The termination indicator (dots 1-2-4-5-6) ⠻

The following steps outline how to write the principal square root of 4 in Nemeth Code:

1. Radical symbol (dots 3-4-5) $(\sqrt{\ \ })$ ⠜
2. Four (dots 2-5-6) ⠲
3. Termination indicator (dots 1-2-4-5-6) ⠻

$\sqrt{4}$

⠜⠲⠻

When writing a radical with an index, you follow these simple steps. You would braille:

1. The index-of-radical indicator (dots 1-2-6) ⠣

2. The index of the radical
3. The radical symbol (dots 3-4-5) ⠜
4. The radicand, value inside/under a radical symbol, which you want to find the root of
5. The termination indicator (dots 1-2-4-5-6) ⠻

The following steps outline how to write the cube root of 27 in Nemeth Code:

1. Index-of-radical indicator (dots 1-2-6) ⠣
2. Three (dots 2-5) ⠒
3. Radical symbol (dots 3-4-5) ⠜
4. Twenty-seven (dots 2-3, dots 2-3-5-6) ⠆⠶
5. Termination indicator (dots 1-2-4-5-6) ⠻

$$\sqrt[3]{27}$$

⠣⠒⠜⠆⠶⠻

Basic Rules for Multiplying Radicals

For any nonnegative real numbers a and b, the square root of a times (multiplication dot) the square root of b equals the square root of a b.

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$$

For any nonnegative real numbers a and b, and any natural number index k, the kth root of a times (multiplication dot) the kth root of b equals the kth root of a b. If k is an odd index, then the kth root of a times the kth root of b equals the kth root of a b for all real numbers a and b.

$$\sqrt[k]{a} \cdot \sqrt[k]{b} = \sqrt[k]{ab}$$

This theorem is used extensively when multiplying and simplifying radical expressions, as shown below. Follow the same conventions for reading and spacing as we did in Lessons 1 to 3.

Examples of Multiplying Radicals

In the examples below, when you hear the word times, only use a multiplication dot when specifically indicated.

1. The square root of 7 end root times the square root of 2 end root (without using the times sign) equals the square root of 7 end root times (multiplication dot) the square root of 2 end root.

$$\sqrt{7}\,\sqrt{2} = \sqrt{7} \cdot \sqrt{2}$$

⠲⠂ ⠲⠆ ⠀ ⠲⠶⠦⠲⠆⠒ ⠀ ⠀⠐⠶ ⠀ ⠀⠲⠶⠦⠐⠄⠲⠆⠒

2. The cube root of 7 end root times (multiplication dot) the cube root of 2 end root equals the cube root of 7 end root times the cube root of 2 end root (without using the times sign).

$$\sqrt[3]{7} \cdot \sqrt[3]{2} = \sqrt[3]{7}\sqrt[3]{2}$$

⠲⠆ ⠲⠆ ⠀ ⠠⠆⠖⠲⠶⠦⠲⠆⠠⠆⠖⠲⠆⠒ ⠀ ⠐⠶ ⠀ ⠠⠆⠖⠲⠶⠦⠲⠆⠠⠆⠖⠲⠆⠒

Activity Time for Multiplying Radicals

Write the problems from Examples 1 to 2.

1. The square root of 7 end root times the square root of 2 end root (without using the times sign) equals the square root of 7 end root times (multiplication dot) the square root of 2 end root.
2. The cube root of 7 end root times (multiplication dot) the cube root of 2 end root equals the cube root of 7 end root times the cube root of 2 end root (without using the times sign).

Examples of Multiplying and Simplifying Square Roots

1. The square root of x plus two end root times the square root of x minus two end root equals the square root of open parenthesis x plus two close parenthesis open parenthesis x minus two close parenthesis end root equals the square root of x squared minus four end root.

$$\sqrt{x+2}\sqrt{x-2} = \sqrt{(x+2)(x-2)} = \sqrt{x^2 - 4}$$

⠲⠂ ⠲⠆ ⠀ ⠲⠷⠭⠖⠆⠒⠲⠷⠭⠤⠆⠒ ⠀ ⠐⠶ ⠀ ⠲⠷⠦⠷⠭⠖⠆⠾⠦⠷⠭⠤⠆⠾⠒⠦⠒
⠀⠀⠀⠀ ⠐⠶ ⠀ ⠲⠷⠭⠘⠆⠤⠲⠒

2. The square root of twenty end root equals the square root of four times (multiplication dot) five end root equals the square root of four end root times (multiplication dot) the square root of five end root equals two square root of five end root.

$$\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$$

⠫⠄⠼⠃⠚⠻⠀⠨⠅⠫⠄⠼⠙⠡⠑⠻⠀⠨⠅⠀⠫⠄⠼⠙⠻⠡⠫⠄⠼⠑⠻⠀⠨⠅⠀⠼⠃⠫⠄⠼⠑⠻

⠀⠀⠨⠅⠀⠼⠃⠫⠄⠼⠑⠻

3. The square root of twelve z plus twelve end root equals the square root of four open parenthesis three z plus three close parenthesis end root equals the square root of two squared open parenthesis three z plus three close parenthesis end root equals two square root of three z plus three end root.

$$\sqrt{12z + 12} = \sqrt{4(3z + 3)} = \sqrt{2^2(3z + 3)} = 2\sqrt{3z + 3}$$

⠫⠄⠼⠁⠃⠰⠵⠬⠼⠁⠃⠻⠀⠨⠅⠀⠫⠄⠼⠙⠷⠼⠉⠰⠵⠬⠼⠉⠾⠻

⠀⠀⠨⠅⠀⠫⠄⠼⠃⠘⠼⠃⠐⠷⠼⠉⠰⠵⠬⠼⠉⠾⠻⠀⠨⠅⠀⠼⠃⠫⠄⠼⠉⠰⠵⠬⠼⠉⠻

Activity Time for Multiplying and Simplifying Square Roots

Write the problems from Examples 1 to 3.

1. The square root of x plus two end root times the square root of x minus two end root equals the square root of open parenthesis x plus two close parenthesis open parenthesis x minus two close parenthesis end root equals the square root of x squared minus four end root.
2. The square root of twenty end root equals the square root of four times (multiplication dot) five end root equals the square root of four end root times (multiplication dot) the square root of five end root equals two square root of five end root.
3. The square root of twelve z plus twelve end root equals the square root of four open parenthesis three z plus three close parenthesis end root equals the square root of two squared open parenthesis three z plus three close parenthesis end root equals two square root of three z plus three end root.

Examples of Multiplying and Simplifying Indexed Radicals

1. The cube root of four end root times the cube root of five end root equals the cube root of four times (multiplication dot) five end root equals the cube root of twenty end root.

$$\sqrt[3]{4}\,\sqrt[3]{5} = \sqrt[3]{4 \cdot 5} = \sqrt[3]{20}$$

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⠨⠅ ⠨⠡⠲⠆⠴⠴⠴⠨⠂

2. Three cube root of twenty-five end root times (multiplication dot) two cube root of five end root equals six cube root of twenty-five times (multiplication dot) five end root equals six cube root of one hundred twenty-five end root equals six times (multiplication dot) five equals thirty.

$$3\sqrt[3]{25} \cdot 2\sqrt[3]{5} = 6\sqrt[3]{25 \cdot 5} = 6\sqrt[3]{125} = 6 \cdot 5 = 30$$

⠼⠒⠀⠨⠡⠲⠆⠢⠴⠴⠴⠂⠸⠲⠆⠀⠼⠆⠀⠨⠡⠲⠢⠴⠴⠴⠨⠂⠀⠨⠅⠀⠼⠖⠀⠨⠡⠲⠆⠢⠸⠲⠢⠴⠴⠴⠂⠴⠦

⠨⠅⠀⠼⠖⠀⠨⠡⠲⠂⠆⠢⠴⠴⠴⠨⠂⠀⠨⠅⠀⠼⠖⠸⠲⠢⠀⠨⠅⠀⠼⠒⠴

3. The fifth root of sixteen end root times (multiplication dot) the fifth root of four end root equals the fifth root of two to the fourth power end root times (multiplication dot) the fifth root of two squared end root equals the fifth root of two to the fourth power times (multiplication dot) two squared end root.

$$\sqrt[5]{16} \cdot \sqrt[5]{4} = \sqrt[5]{2^4} \cdot \sqrt[5]{2^2} = \sqrt[5]{2^4 \cdot 2^2}$$

⠲⠢⠦ ⠲⠢⠔ ⠨⠡⠢⠂⠖⠴⠸⠲⠲⠢⠲⠴

⠨⠅ ⠨⠡⠢⠲⠆�⠴⠸⠲⠨⠡⠢⠲⠆�⠴

⠨⠅ ⠨⠡⠢⠲⠆�⠸⠲⠲⠆�⠴

4. The fifth root of two to the fourth power times (multiplication dot) two squared end root equals the fifth root of two to the sixth power end root equals two fifth root of two end root.

$$\sqrt[5]{2^4 \cdot 2^2} = \sqrt[5]{2^6} = 2\sqrt[5]{2}$$

⠰⠤ ⠰⠤⠤ ⠀ ⠤⠰⠤⠤⠤⠤ ⠤⠤ ⠤⠤⠤⠤ ⠤⠤ ⠤⠤ ⠀⠀ ⠒⠒ ⠀ ⠤⠰⠤⠤⠤ ⠤⠤ ⠤⠤

⠀⠀⠀⠒⠒⠀⠀⠰⠤⠤⠰⠤⠤⠤⠤⠤⠤

5. The fifth root of y cubed end root times (multiplication dot) the fifth root of y to the fourth power end root equals the fifth root of y cubed times (multiplication dot) y to the fourth power end root equals the fifth root of y to the seventh power end root equals y fifth root of y squared end root.

$$\sqrt[5]{y^3} \cdot \sqrt[5]{y^4} = \sqrt[5]{y^3 \cdot y^4} = \sqrt[5]{y^7} = y\sqrt[5]{y^2}$$

⠰⠤ ⠰⠤⠤ ⠀ ⠤⠰⠤⠤⠤ ⠤⠤ ⠤⠤⠤⠰⠤⠤⠤⠤⠤⠤ ⠤⠤

⠀⠀⠀⠒⠒⠀⠀⠤⠰⠤⠤⠤ ⠤⠤ ⠤⠤⠤⠤ ⠤⠤ ⠀ ⠒⠒ ⠀ ⠤⠰⠤⠤ ⠤⠤ ⠤⠤

⠀⠀⠀⠒⠒⠀⠀⠤⠰⠤⠰⠤⠤ ⠤⠤ ⠤⠤

Activity Time for Multiplying and Simplifying Indexed Roots

Write the problems from Examples 1 to 5.

1. The cube root of four end root times the cube root of five end root equals the cube root of four times (multiplication dot) five end root equals the cube root of twenty end root.
2. Three cube root of twenty-five end root times (multiplication dot) two cube root of five end root equals six cube root of twenty-five times (multiplication dot) five end root equals six cube root of one hundred twenty-five end root equals six times (multiplication dot) five equals thirty.
3. The fifth root of sixteen end root times (multiplication dot) the fifth root of four end root equals the fifth root of two to the fourth power end root times (multiplication dot) the fifth root of two squared end root equals the fifth root of two to the fourth power times (multiplication dot) two squared end root.
4. The fifth root of two to the fourth power times (multiplication dot) two squared end root equals the fifth root of two to the sixth power end root equals two fifth root of two end root.
5. The fifth root of y cubed end root times (multiplication dot) the fifth root of y to the fourth power end root equals the fifth root of y cubed times (multiplication dot) y to the fourth power end root equals the fifth root of y to the seventh power end root equals y fifth root of y squared end root.